

Non-Riemannian geometrical optics in QED

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Abstract

A non-minimal photon-torsion axial coupling in the quantum electrodynamics (QED) framework is considered. The geometrical optics in Riemannian-Cartan spacetime is considering and a plane wave expansion of the electromagnetic vector potential is considered leading to a set of the equations for the ray congruence. Since we are interested mainly on the torsion effects in this first report we just consider the Riemann-flat case composed of the Minkowskian spacetime with torsion. It is also shown that in torsionic de Sitter background the vacuum polarisation does alter the propagation of individual photons, an effect which is absent in Riemannian spaces.

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1 Introduction

A renewed interest in nonlinear electrodynamics has been recent put forward in the papers of Novello and his group [1, 2] specially concerning the investigation of a Born-Infeld electrodynamics in the context of general relativity (GR). The interesting feature of these non-linear electrodynamics and some Chern-Simons electrodynamics is the fact as shown previously by de Sabbata and Gasperini [3, 4] which consider a perturbative approach to QED and ends up with a generalized Maxwell equation with totally skew torsion. The photon-torsion perturbation obtained by this perturbative calculation allows the production of virtual pairs which is the vacuum polarization effect or QED. In this paper we consider the non-minimal extension of QED, given previously in Riemannian spacetime by Drummond and Hathrell [5] to Riemann-Cartan geometry. We should like to stress here that the photon-torsion coupling considered in the paper comes from the interaction of a Riemann-Cartan tensor to the electromagnetic field tensor in the Lagrangean action term of the type $R_{ijkl}F^{ij}F^{kl}$ where $i, j = 0, 1, 2, 3$. Therefore here we do not have the usual problems of the noninteraction between photons and torsion as appears in the usual Maxwell electrodynamics [6]. The plan of the paper is as follows : In section II we consider the formulation of the Riemann-Cartan (RC) nonlinear electrodynamics and show that in de Sitter case the vacuum polarisation does alter the propagation of individual photons. In section III the Riemann-flat case is presented and geometrical optics in non-Riemannian spacetime along with ray equations are given. Section IV deals the conclusions and discussions.

2 De Sitter torsioned spacetime and nonlinear electrodynamics

Since the torsion effects are in general too weak as can be seen from recent evaluations with K mesons (kaons) [7] which yields $10^{-32} GeV$, we consider throughout the paper that second order effects on torsion can be dropped out from the formulas of electrodynamics and curvature. In this section we consider a simple cosmological application concerning the nonlinear electrodynamics in de Sitter spacetime background. The Lagrangean used in this

paper is obtained from the work of Drummond et al [5]

$$W = \frac{1}{m^2} \int d^4x (-g)^{\frac{1}{2}} (a R F^{ij} F_{ij} + b R_{ik} F^{il} F^k_l + c R_{ijkl} F^{ij} F^{kl} + d D_i F^{ij} D_k F^k_j) \quad (1)$$

The constant values a, b, c, d may be obtained by means of the conventional Feynman diagram techniques [5]. The field equations obtained are [5]

$$D_i F^{ik} + \frac{1}{m_e^2} D_i [4a R F^{ik} + 2b (R^i_l F^{lk} - R^k_l F^{li}) + 4c R^{ik}_{lr} F^{lr}] = 0 \quad (2)$$

$$D_i F^{jk} + D_j F^{ki} + D_k F^{ij} = 0 \quad (3)$$

where D_i is the Riemannian covariant derivative, $F^{ij} = \partial^i A^j - \partial^j A^i$ is the electromagnetic field tensor non-minimally coupled to gravity, and A^i is the electromagnetic vector potential, R is the Riemannian Ricci scalar, R_{ik} is the Ricci tensor and R_{ijkl} is the Riemann tensor. Before we apply it to the de Sitter model, let us consider several simplifications. The first concerns the fact that the photon is treated as a test particle, and the second considers simplifications on the torsion field. The Riemann-Cartan curvature tensor is given by

$$R^{*ij}_{kl} = R^{ij}_{kl} + D^i K^j_{kl} - D^j K^i_{kl} + [K^i, K^j]_{kl} \quad (4)$$

the last term here shall be dropped since we are just considering the first order terms on the contortion tensor. Quantities with an upper asterisk represent RC geometrical quantities. We also consider only the axial part of the contortion tensor K_{ijk} in the form

$$K^i = \epsilon^{ijkl} K_{jkl} \quad (5)$$

to simplify equation (2) we consider the expression for the Ricci tensor as

$$R^{*i}_k = R^i_k - \epsilon^i_{klm} D^{[l} K^{m]} \quad (6)$$

where ϵ^i_{klm} is the totally skew symmetric Levi-Civita symbol. By considering the axial torsion as coming from a dilaton field ϕ one obtains

$$K^i = D^i \phi \quad (7)$$

Substitution of expression (7) into formula (6) yields

$$\partial^{[l} K^{m]} = 0 \quad (8)$$

Thus expression (6) reduces to

$$R^{*i}{}_k = R^i{}_k \quad (9)$$

Therefore note that in the Riemann-flat case we shall be considering in the next section, $R_{ijkl} = 0$ and $R^{*i}{}_k = 0$ which strongly simplifies the Maxwell type equation. In this section the de Sitter curvature

$$R_{ijkl} = K(g_{ik}g_{jl} - g_{il}g_{jk}) \quad (10)$$

contraction of this expression yields

$$R = K \quad (11)$$

and substitution of these contractions into the Maxwell-like equation one obtains

$$(1 + 2\xi^2 K) D_i F^i{}_k = \epsilon_{klmn} D^i K^n D_i F^{lm} \quad (12)$$

Here $\xi^2 = \frac{\alpha}{90\pi m_e^2}$ where m_e is the electron mass and α is the fine structure constant. This equation shows that the vacuum polarisation alters the photon propagation in de Sitter spacetime with torsion. This result may provide interesting applications in cosmology such as in the study of optical activity in cosmologies with torsion such as in Kalb-Ramond cosmology [8].

3 Riemann-flat nonlinear torsionic electrodynamics

In this section we shall be concerned with the application of nonlinear electrodynamics with torsion in the Riemann-flat case, where the Riemann curvature tensor vanishes. In particular we shall investigate the non-Riemannian geometrical optics associated with that. Earlier L.L.Smalley [9] have investigated the extension of Riemannian to non-Riemannian RC geometrical optics in the usual electrodynamics, nevertheless in his approach was not clear if the torsion really could coupled with photon. Since the metric considered here is

the Minkowski metric η_{ij} we note that the Riemannian Christoffel connection vanishes and the Riemannian derivative operator D_k shall be replaced in this section by the partial derivative operator ∂_k . With these simplifications the Maxwell like equation (2) becomes

$$\partial_i F^{ij} + \xi^2 R^{ij}_{kl} \partial_i F^{kl} = 0 \quad (13)$$

which reduces to

$$\partial_i F^{ik} + \xi^2 [\epsilon^k_{jlm} \partial^i K^m - \epsilon^i_{jlm} \partial^k K^m] \partial_i F^{jl} = 0 \quad (14)$$

we may also note that when the contortion is parallel transported in the last section the equations reduce to the usual Maxwell equation

$$D_i F^{il} = 0 \quad (15)$$

Since we are considering the non-minimal coupling the Lorentz condition on the vector potential is given by

$$\partial_i A^i = 0 \quad (16)$$

with this usual Lorentz condition substituted into the Maxwell-like equation one obtains the wave equation for the vector electromagnetic potential as

$$\square A^i + \xi^2 [\epsilon^k_{jlm} \partial^i K^m - \epsilon^i_{jlm} \partial^k K^m] \partial_k \partial^j A^l = 0 \quad (17)$$

Now to obtain the equations for the Riemann-Cartan geometrical optics based on the nonlinear electrodynamics considered here we just consider the plane wave expansion

$$A^i = \text{Re}[(a^i + \epsilon b^i + c^i \epsilon^2 + \dots) e^{i \frac{\theta}{\epsilon}}] \quad (18)$$

Substitution of this plane wave expansion into the Lorentz gauge condition one obtains the usual orthogonality condition between the wave vector $k_i = \partial_i \theta$ and the amplitude a^i up to the lowest order given by

$$k^i a_i = 0 \quad (19)$$

note that by considering the complex polarisation given by $a^i = a f_i$ expression (19) reduces to

$$k_i f^i = 0 \quad (20)$$

$$\partial_i k^i = \frac{\xi^2}{a^2} \epsilon^{ijkl} a_i b_k k_j{}^{,r} \partial_r K_l \quad (21)$$

this equation describes the expansion or focusing of the ray congruence and the influence of contortion inhomogeneity in it. The last equation is

$$k^i k_i = -\frac{\xi^2}{a^2} \epsilon^{ijkl} a_i [a_j b_k - a_k b_j] k^r \partial_r K_l \quad (22)$$

note however that the RHS of (22) vanishes identically due to the symmetry in the product of the a^n vector contracting with the skew Levi-Civita symbol, and finally we are left with the null vector condition $k_j k^j = 0$.

4 Discussion and conclusions

The geometrical optics discussed in the last section allows us to build models to test torsion effects on gravitational optical phenomena such as gravitational lensing and optical activity. Besides the geometrical optics investigated in the last section could be reproduced in the case of de Sitter cosmology. This approach can be considered in near future.

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